

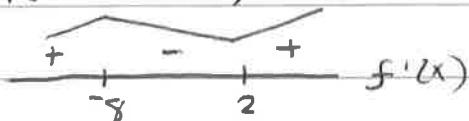
Extrema Worksheet

1. $f(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 5$

$f'(x) = x^2 + 6x - 16$

$0 = (x + 8)(x - 2)$

CN: $x = -8, 2$

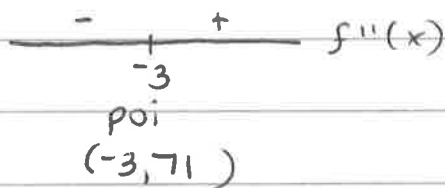


max $(-8, 154.3)$ min $(2, -12.3)$

$f''(x) = 2x + 6$

$0 = 2(x + 3)$

CN: $x = -3$



a) $(-\infty, -8) \cup (2, \infty)$

b) $(-8, 2)$

c) $(-3, \infty)$

d) $(-\infty, -3)$

e) $(-8, 154.3)$

f) $(2, -12.3)$

g) $(-3, 71)$

h) $(0, 5)$

i) see graph

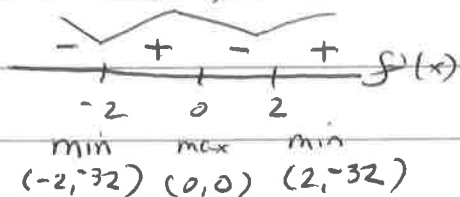
2. $f(x) = 2x^4 - 16x^2$

$f'(x) = 8x^3 - 32x$

$0 = 8x(x^2 - 4)$

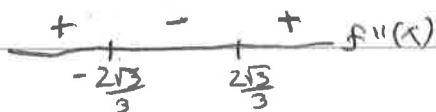
$0 = 8x(x + 2)(x - 2)$

CN: $x = 0, -2, 2$



$f''(x) = 24x^2 - 32$

$0 = 8(3x^2 - 4)$ $x = \pm \frac{2}{3}$



a) $(-2, 0) \cup (2, \infty)$

b) $(-\infty, -2) \cup (0, 2)$

c) $(-\infty, -\frac{2\sqrt{3}}{3}) \cup (\frac{2\sqrt{3}}{3}, \infty)$

d) $(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3})$

e) $(0, 0)$

f) $(-2, -32)$ & $(2, -32)$

g) $(-\frac{2\sqrt{3}}{3}, -17.8)$ & $(\frac{2\sqrt{3}}{3}, -17.8)$

h) $(0, 0)$

i) see graph

$$3. f(x) = \frac{x+2}{x-9}$$

$$f'(x) = \frac{(1)(x-9) - (x+2)(1)}{(x-9)^2}$$

$$0 = \frac{x-9 \cancel{x-2}}{(x-9)^2}$$

$$0 = \frac{-11}{(x-9)^2}$$

$$\text{CN: } x=9$$

$$\begin{array}{c} - \quad - \\ | \\ 9 \end{array} \quad f'(x)$$

$$f'(x) = -11(x-9)^{-2}$$

$$f''(x) = 22(x-9)^{-3}$$

$$0 = \frac{22}{(x-9)^3}$$

$$\text{CN: } x=9$$

$$\begin{array}{c} - \quad + \\ | \\ 9 \end{array} \quad f''(x)$$

a) none

b) $(-\infty, 9) \cup (9, \infty)$

c) $(9, \infty)$

d) $(-\infty, 9)$

e) none

f) none

g) none

h) $(0, -2/9)$

i) see graph

$$4. f(x) = \frac{9}{5} x^{5/3} - 12x$$

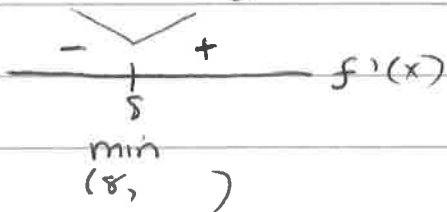
$$f'(x) = 3x^{2/3} - 12$$

$$0 = 3x^{2/3} - 12$$

$$12 = 3x^{2/3}$$

$$(4)^{3/2} (x^{2/3})^{3/2}$$

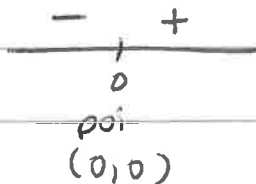
$$CN: x = 8$$



$$f''(x) = 2x^{-1/3}$$

$$0 = \frac{2}{x^{1/3}}$$

$$CN: x = 0$$



$$a) (8, \infty)$$

$$b) (-\infty, 8)$$

$$c) (0, \infty)$$

$$d) (-\infty, 0)$$

$$e) \text{ none}$$

$$f) (8, -38.4)$$

$$g) (0, 0)$$

$$h) (0, 0)$$

$$i) \text{ see graph}$$

$$5. f(x) = \frac{2}{x^2 - 2x + 1}$$

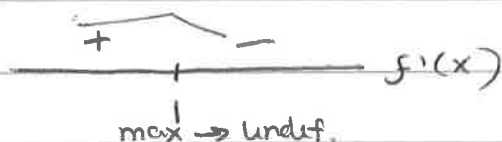
$$f(x) = 2(x^2 - 2x + 1)^{-1}$$

$$f'(x) = -2(x^2 - 2x + 1)^{-2}(2x - 2)$$

$$0 = \frac{-2(2x - 2)}{(x^2 - 2x + 1)^2} \rightarrow ((x - 1)(x - 1))^2$$

$$0 = -2(2x - 2)$$

$$\text{CN: } x = 1$$



$$f(x) = (-4x + 4)(x^2 - 2x + 1)^{-2}$$

$$f''(x) = (-4)(x^2 - 2x + 1)^{-2} + (-4x + 4)(-2)(x^2 - 2x + 1)^{-3}(2x - 2)$$

$$0 = -2(x^2 - 2x + 1)^{-3} [2(x^2 - 2x + 1) + (-4x + 4)(2x - 2)]$$

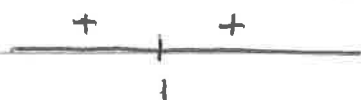
$$0 = -2(x^2 - 2x + 1)^{-3} [2x^2 - 4x + 2 + -8x^2 + 8x + 8x - 8]$$

$$0 = -2(x^2 - 2x + 1)^{-3} [-6x^2 + 12x - 6]$$

$$0 = 12(x^2 - 2x + 1)$$

$$(x^2 - 2x + 1)^3 \rightarrow \text{CN: } x = 1$$

$$0 = 12(x - 1)(x - 1)$$



a) $(-\infty, 1)$

b) $(1, \infty)$

c) $(-\infty, 1) \cup (1, \infty)$

d) none

e) none

f) none

g) none

h) $(0, 2)$

i) see graph

$$6. f(x) = 2x^3 - 9x^2 \quad [-1, 5]$$

$$f'(x) = 6x^2 - 18x$$

$$f(0) = 0$$

$$0 = 6x(x-3)$$

$$f(3) = -27 \rightarrow \text{abs. min}$$

$$\text{CN: } x = 0, 3$$

$$f(-1) = -11$$

$$f(5) = 25 \rightarrow \text{abs. max}$$

$$7. f(x) = x^4 - 4x^3 \quad [-1, 4]$$

$$f'(x) = 4x^3 - 12x^2$$

$$f(0) = 0$$

$$0 = 4x^2(x-3)$$

$$f(3) = -27 \rightarrow \text{abs. min}$$

$$\text{CN: } x = 0, 3$$

$$f(-1) = 5 \rightarrow \text{abs. max}$$

$$f(4) = 0$$

$$8. f(x) = 2x^3 - 6x \quad [-3, 1.5]$$

$$f'(x) = 6x^2 - 6$$

$$f(-1) = 4 \rightarrow \text{abs. max}$$

$$0 = 6(x^2 - 1)$$

$$f(1) = -4$$

$$0 = 6(x+1)(x-1)$$

$$f(-3) = -36 \rightarrow \text{abs. min}$$

$$\text{CN: } x = -1, 1$$

$$f(1.5) = -2.25$$

$$9. f(x) = \frac{1}{3}x^3 - 9x + 6 \quad [-5, 7]$$

$$f'(x) = x^2 - 9$$

$$f(-3) = 24$$

$$0 = (x+3)(x-3)$$

$$f(3) = -12 \rightarrow \text{abs. min}$$

$$\text{CN: } x = -3, 3$$

$$f(-5) = 9.333$$

$$f(7) = 57.333 \quad * \text{ no abs. max}$$

$$10. f(x) = x^4 - 12x^3 + 48x^2 - 64x \quad [0, 5]$$

$$f'(x) = 4x^3 - 36x^2 + 96x - 64$$

$$0 = 4(x^3 - 9x^2 + 24x - 16)$$

$$0 = 4[x^2(x-9) + 8(3x-2)]$$

$$11. f(x) = x^{1/3} + 2x - 3 \quad [-2, 2]$$

$$f'(x) = \frac{1}{3}x^{-2/3} + 2$$

$$f(-2) = -8.260 \rightarrow \text{abs. min}$$

$$0 = \frac{1}{3}x^{2/3} + 2$$

$$f(2) = 2.260$$

$$-2 = \frac{1}{3}x^{2/3}$$

*no abs. max

$$-6x^{2/3} = 1$$

$$(x^{2/3})^{3/2} = \left(-\frac{1}{6}\right)^{3/2}$$

$$\text{CN: } x = 0$$

$\hookrightarrow$ not a max/min

$$12. f(x) = x^2 - 2x \quad [0, 2]$$

$$f'(x) = 2x - 2$$

$$0 = 2(x - 1)$$

$$\text{CN: } x = 1$$

$$\begin{array}{c} \diagup \quad \diagdown \\ - \quad + \\ | \\ 1 \end{array} \quad f'(x)$$

$$13. f(x) = x^2 - 3x + 2 \quad [1, 2]$$

$$f'(x) = 2x - 3$$

$$0 = 2x - 3$$

$$\text{CN: } x = \frac{3}{2}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ - \quad + \\ | \\ \frac{3}{2} \end{array}$$

$$14. f(x) = (x-1)(x-2)(x-3) \quad [1, 3]$$

$$f(x) = (x-1)(x^2 - 5x + 6)$$

$$f(x) = x^3 - 5x^2 + 6x - x^2 + 5x - 6$$

$$f(x) = x^3 - 6x^2 + 11x - 6$$

$$f'(x) = 3x^2 - 12x + 11$$

15. $f(x) = x^{2/3} - 1$ $[-8, 8]$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$21. f(x) = x^{2/3} \quad [0, 1]$$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$f(0) = 0$$

$$f(1) = 1$$

$$m = \frac{1-0}{1-0} = 1$$

$$1 = \frac{2}{3x^{1/3}}$$

$$3x^{1/3} = 2$$

$$(x^{1/3})^3 = \left(\frac{2}{3}\right)^3$$

$$x = \frac{8}{27}$$

$$22. f(x) = \frac{x+1}{x} \quad \left[\frac{1}{2}, 2\right]$$

$$f'(x) = \frac{(1)(x) - (x+1)(1)}{x^2}$$

$$f\left(\frac{1}{2}\right) = 3$$

$$f(2) = \frac{3}{2}$$

$$f'(x) = \frac{-1}{x^2}$$

$$m = \frac{\frac{3}{2} - 3}{2 - \frac{1}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$$

$$-1 = \frac{-1}{x^2}$$

$$+x^2 = 1$$

$$x = \pm 1 \rightarrow x = 1$$